INTERACTIVE RESOURCES FOR AN ACTIVE DESCRIPTIVE GEOMETRY LEARNING

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The author intends to promote a debate on the need to reorient Descriptive Geometry teaching practices in Portuguese High Schools, so that better responses to the present requirements are achieved, aiming so to improve students' capacities to understand and represent geometric concepts and its relations in three-dimensional space, through a better comprehension of what they represent.

To exemplify the benefits of exploring digital tools with educational purposes, the author presents some interactive resources created with GeoGebra, Rhinoceros and Grasshopper to complement Descriptive Geometry teaching, in order to assist the learning process from the student's perspective and illustrate the potentialities these software can offer to construct educational resources and expand teaching practices.

Keywords: Learning Styles, Descriptive Geometry Teaching, Dynamic Geometry Software, 3D Modelling software, Grasshopper.

Learning styles

Several studies report that some students react best to certain teaching methods than others and that learning is more elective and everlasting when subjects are approached in different ways and, particularly, when teaching strategies address different knowledge dimensions. The investigations of Richard Felder and Linda Silverman and, later, Richard Felder and Barbara Soloman with engineering teachers and students led them to define a classification scheme to understand the "preferences and tendencies students have for certain ways of taking in and processing information and responding to different instructional environments" (Felder, 2010, p.4). According to each personality, the reaction to a particular teaching strategy or even the subject involved, different students receive and process the information differently from others, the authors concluded. Aiming to recognize the preferences of the students in perceiving the information, they defined the following learning styles, which do not necessarily correspond to opposite categories, but to intuitive skills or intelligence levels that may even coexist in the same person:

> "Active and Reflective Learners: Active students learn best if given the opportunity to act and interact, that is, by doing something and discussing, applying and explaining it to other colleagues. Reflective students learn best if they are given the opportunity to think over and ponder, calmly and introspectively, upon the information received.
>
> Sensing and Intuitive Learners: Sensing students like to learn facts, data, principles and theories with a direct relation to situations occurring in real life, while intuitive students prefer to discover possible relationships between these facts (…).
>
> Visual and Verbal Learners: Visual students learn best if they can visualize images, demonstrations, graphs, drawings, pictures, movies, timelines, etc., while verbal students learn best through discussions and verbal explanations (anyone, in fact, will learn best if the subject is presented visually, because everyone is mostly a visual learner).

> Sequential and Global Learners: Sequential students prefer to learn subjects gradually and orderly sequenced, so they can find solutions to the problem, while global students prefer to absorb them almost randomly (…) so that the whole problem makes sense." (Viana, 2014, p.48).

Thorough investigations led the authors to the conclusion that traditional teaching methods do not address the majority of student's population, because they tend to privilege a passive behaviour from students, mostly focused on sitting steadily, taking notes and memorizing the information that in class is regularly presented in written form, through school books or in the classroom board.

Houghton (2004) also points out the emergence of active teaching methodologies in order to sustain a deeper learning from the students' perspective, as opposed to more traditional teaching methods, that support attitudes short in creativity and devoid of interaction. The educational strategies directed towards a deeper learning must necessarily include an active role for the students in a way they are able to understand the subjects taught by interacting with, sidestepping from the surface learning traditional teaching methods endorse. As Houghton points out, working abstracts concepts without a strong understanding of its basic principles will only contribute to a shallow and frustrating understanding of concepts (in his own words, to a surface learning), in which students memorize solutions for the complex problems that they are not able to understand.

Although the multiple learning styles theory in itself is open to debate, because some refute it as irrelevant (for lacking scientific data to certify them properly) while others consider it valid, it may be regarded as basis for a renewed conceptual framework that, according to "A good inventory to identify students learning styles" (2015), may help teachers "in understanding how students learn" inspiring "the pedagogical discussions surrounding learning strategies and instructional modes".

Teaching strategies for an active learning

> "In order to continue to use a new technology for doing mathematics we have to learn to use it in ways which transform mathematical activity, enabling us to do things which would not previously have been possible." (Sutherland, 2005, p.47)

To assist teachers to better adapt its teaching methodologies to the different learning styles found in the classroom, Felder and Silverman proposed some "Teaching Techniques to Address All Learning Styles" (Felder & Silverman, 1988, p.680). For the sake of conciseness, we will focus solely in one of the teaching techniques that can addresses to various learning styles and that, supported by the majority of subsequent investigations on these matters, best suits our train of thought: "Use computer-assisted instruction" (Felder & Silverman, 1988, p. 680).

As we will try to explain, the proficient exploration of digital technologies (and particularly, for the subject depicted, specific software dealing with geometric concepts and 3D modelling) can complement everyday educational practices, through innovative methods with a huge potential for broadening teaching methodologies that may inspire more consisting learning outcomes. If conceived to complement traditional teaching methods and well oriented, the use of computers in the educational context can attend to many different learning styles, not only for its visual attractiveness and interactive possibilities (which may increase the motivation for active, reflective, visual and sensing students to learn) but also because they can effectively support teachers practices.

Jaime Carvalho e Silva (2014, p. 3) sustains that schools should offer students the new tools that our technology in rapid evolution brings to our everyday life and the professional activities that "students will find someday in their adult life". Joel Klein, cited by Carlo Rotella (2013), states that if the use of the computer in the classroom is "not transformative, it's not worth it.", for it "can only make the hoped-for difference in how and what students learn if teachers come up with new ways to use it." On the same train of thought, Michael de Villiers (2006) referring the many pitfalls that befall in the introduction of computers in the teaching of Mathematics, emphasises the need for the "development of new skills" (2006, p.46) for the teachers that introduce them in the classroom. Reinforcing this inevitability, the International Society for Technology in Education (2014) states, as one of the standards and performance indicators, the need for teachers to develop "technology-enriched learning environments (…) and personalize learning activities to address students' diverse learning styles, working strategies, and abilities using digital tools and resources".

In conclusion, it is advisable that, for the majority of the school subjects, teachers explore digital technologies recurrently as pedagogical tools, taking full advantage of its possibilities, whenever possible, in innovative ways so that the teaching experience can be meaningful and, desirably, long-lasting in students' memories.

Digital educational tools should be explored in the classroom for the positive effects they offer, particularly when intended to give students the possibility to interact and to experience something pedagogically significant and different from the traditional educational context and from a mere "extension of paper and pencil geometry", that according to Sutherland (2005, p.4), many teachers often do, when using computers in the classroom. Quite the opposite, the exploration of computer-based technologies in this context should foremost assist students in their educational experience, while the teacher provides guidance whenever necessary to explain what is to be retained of it. But, as Michael de Villiers mentions:

> "Dynamic Geometry cannot offer a magical panacea for learning Geometry (…) simply by staring at the beautiful, moving pictures on the screen. Unless the learner or student critically engages or is carefully guided to observe and examine what is happening on the screen, very little learning may actually be taking place" (2006, p.48).

Thus the importance of guiding students in computer assisted activities assuring they are not led to think that software is more important than our own reasoning or that learning Geometry is no longer required. Some dynamic Geometry applets can be constructed beforehand so students explore them autonomously. This kind of applets might be very useful from the students' perspective, especially if they are instructed by the teacher to use them properly. It is also advisable that teachers tell students how traditional methods would not accomplish the problem's resolution in the way that software can provide without numerous intermediary steps (that still the teacher should point to as to avoid students to lose track of the underlying geometric concepts).

Computer-based technologies towards an active Geometry learning

> "If we agree that one of the educational goals is to provide youth with basic skills and competencies needed for later professions, what is the role of school Geometry curricula in this respect? Wouldn't one of its important goals be to prepare pupils for this shift of emphasis

> imposed by the use of computerized tools? (…) How can Mathematics educators benefit from the tools available in order to enhance the teaching of Geometry?" (Osta, 1998, pg. 129)

Nowadays, the benefits provided by dynamic Geometry software in the educational context are unquestionable, for they allow teachers to hugely enhance their practice and students, through an "experimental laboratory" (Villiers, 2006, p. 1), to develop their mathematical reasoning with dynamic experiences that can become quite fulfilling, if properly oriented. In "Geometry Turned On!" King & Schattschneider (1997) explore the endless possibilities of dynamic Geometry software for the enhancement of Geometry didactics, providing an experimentation-based educational perspective that would otherwise remain inaccessible by traditional teaching methods.

The benefits brought to the teaching of Mathematics and its learning processes by the introduction of digital tools in the classroom can likewise broaden the horizons for Descriptive Geometry teaching (still taught in a small number of European countries) provided its exploration is oriented in a way it best supports the development of students reasoning and the comprehension of geometric concepts and its relations in space. Dynamic Geometry and 3D Modelling software (Fig. 1) can effectively support teachers' practices and particularly assist students, so they can better understand the underlying concepts depicted at their own learning pace, supporting the process of conceptually operating with geometric concepts and, in doing so, developing their spatial abilities and a desirable enthusiasm in learning Mathematics in a broader sense.

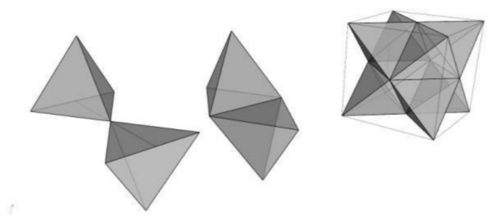


Figure 1 – Examples of non-polyhedra constructed with Rhinoceros

Descriptive Geometry computer-assisted teaching for a positive learning

In a recent interview conducted by Jakobsen & Matthiasen (2014), the participants concluded that students who have the possibility of complementing the practice of Drawing with Descriptive Geometry learning as a graphic representation method are better prepared for the practice of Architecture, Engineering and other areas concerned with representing space and operating with space modelling, because one of the purposes of learning Descriptive Geometry is, foremost, providing students with the ability to "think in space" (Jakobsen & Matthiasen, 2014, p.3).

In many European countries, the Descriptive Geometry subject has been discarded, although it stands as an important subject at the secondary level of education that can address students' spatial reasoning as a requisite of operating with geometric concepts in three-dimensional space. It is our belief that, in Portugal (where Descriptive Geometry is taught in the 10th and 11th grade), the current didactic methodologies and concepts taught should be object of deep adjustments in order to abandon the fierce connections that its teaching still maintains with somehow obsolete proceedings (that are no longer required in the professional context in which students will be integrated); articulate its concepts in deeper connexions with the concepts taught at the same grades in

Mathematics; and desirably, to provide a short period for students to explore themselves Dynamic Geometry and 3D Modelling software. Also, the awareness of students towards the relation between the three-dimensional "reality" and its representation (and vice-versa) should be enhanced and seriously considered the potential benefits that Descriptive Geometry learning can provide towards a more correct interpretation of the graphic representations of geometric objects and, inversely, the ability to represent them in the two-dimensional plane. In this matter, we consider that Descriptive Geometry concepts involving the Axonometric Projection System should be generalized to the students that learn Mathematics at secondary level, in an attempt to optimize the process of coding and decoding graphic representations (Parzysz, 1991, 578).

The importance of learning Geometry and (in the early instruction of every profession that deals with the modelling of space and its representation) Descriptive Geometry, should not be disregarded, for its evident importance in the development of students' geometric reasoning and their spatial abilities. Furthermore, we claim that the inclusion of the Descriptive Geometry in high schools curricula (assumed its necessary adaptation to present day requirements) should be reconsidered, given the significance of a very solid geometric knowledge for operating with three-dimensional concepts every expertise in 3D Modelling software (Fig. 2) must have.

> "The possibilities allowed by any good (…) software expand enormously the possibilities allowed by the ruler and compass on a sheet of paper. (…) This does not mean that this short set of instruments is not efficient from a basic learning perspective. But to go further today, we must use today's means." (Mateus, 2014, pp.54-55)

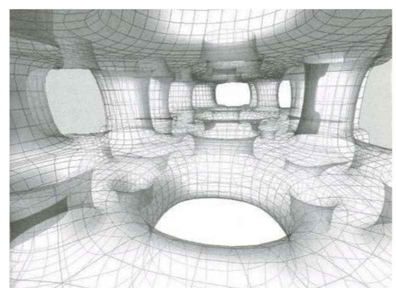
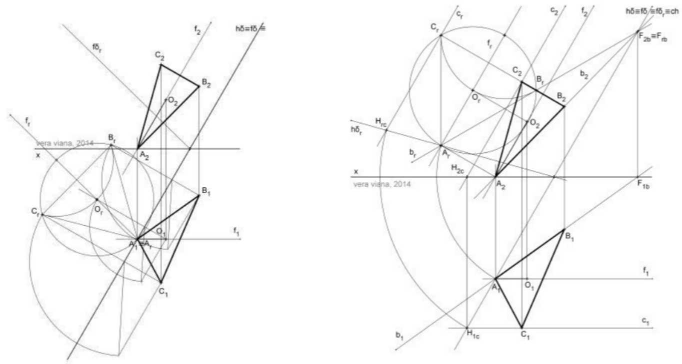


Figure 2 - Taichung Metropolitan Opera House - Toyo Ito.

Interactive GeoGebra resources for an active Descriptive Geometry learning

Taken from the personal website developed since 2007 by the author with the intent to share interactive constructions built with dynamic Geometry software, some applets are presented as possible educational strategies to address the different learning styles. These applets intended to support Descriptive Geometry didactics, are mainly focused in the students' perspective to support the classroom's learning outcomes and complement, in an innovative way, traditional teaching practices. Meant to address learning styles such as those of active, reflective, sensing, intuitive, visual and sequential learners, these applets can, as our teaching experience tells us, stimulate students to better understand the different results obtained from a predetermined situation.

Reflective and sequential students, for instance, may find useful to explore, while studying alone, step-by-step different resolutions of the problem exemplified in Figures 3 and 4. Teachers themselves can complement their educational practice exploring these constructions during classes.



Figures 3 and 4 – Two resolution procedures for a problem involving the orthographic projections of an oblique triangle. www.veraviana.net/diedpassoapassofig.html#EX151PAG129).

All applets are available in the website and some of them in a CD-ROM and/or PEN drive (since 2013) so that teachers can explore them according to their needs in the classroom and students, autonomously, at their own learning rhythm. It is our belief that, if the opportunity to explore these applets is provided in a way students can complement what has been learnt in the classroom, the opportunity to enrich the educational experience from the students' perspective is created in a way that the whole experience may be effectively productive.

With this intent, the example in Fig. 5 allows students to move the points that define a plane, thus understanding how its different locations determines the plane's orientation and, subsequently, its lines of intersection with the projecting planes. This experience may be productive for active, intuitive and visual learners.

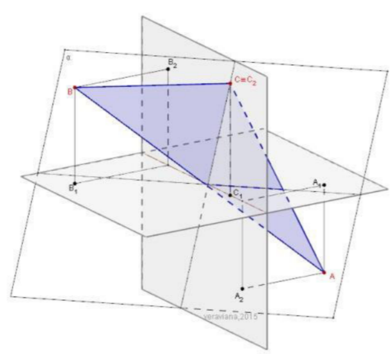


Figure 5 – Interactive 3D Axonometric projection of a triangle and its plane (http://www.veraviana.net/dieddinamicasfg.html#triangulo).

The applets can likewise complement dynamically the practice of teachers in the classroom and given the students the necessary guidance to understand what is shown, so that they are able to "search what remains constant in what is variable" (Xavier & Rebelo, 2001, p.4). The construction in Fig. 6 intends to dynamically demonstrate several possibilities from a specific graphic situation and its purpose is to assist students in a better understanding of the invariant properties of the geometric concepts involved (in this case, the truncation of the cube)

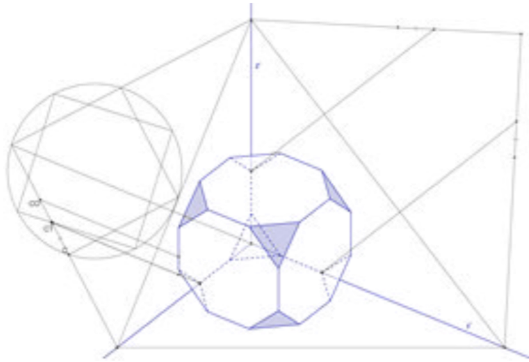


Figure 6 – Axonometric projection of a cube sectioned perpendicularly to the symmetry axis of each vertex (www.veraviana.net/arquimedianos.html#doisarquimedianos).

Most of the applets are available in GeoGebra Books, that distributes them by theme: Descriptive Geometry Applets (step-by-step) - http://tube.geogebra.org/student/bMNeCNYyn and Interactive Descriptive Geometry Applets - http://tube.geogebra.org/student/bcIg8exfg#.

Introducing Polyhedra with Rhinoceros and Grashopper

Our Descriptive Geometry teaching practices demonstrates that students became more interested in the learning process when geometric concepts are presented in different ways. When the subject depicting the orthographic representation of polyhedra is introduced, it is suggested that students manipulate real polyhedral models and, subsequently, infer polyhedral categories through its description and visualization of virtual representations of simple software such as Poly or Stella4D. Gutiérrez (1996, pg. 11) addresses the relevance of this kind of practice as educationally inspiring, because students tend to create mental images of solids in a better way and develop their abilities of mentally rotating similar geometric concepts.

Complementing this experience, 3D modelling software Rhinoceros and its plug-in Grasshopper can reveal themselves quite useful when a more effective visualization display is required to explore concepts dealing with solid geometry. With this intent, Figures 7, 8, 9 and 10 display virtual representations of right pyramids and right prisms that were constructed with the purpose of controlling parametrically the configuration and orientation of the base (or basis) and the solid's height.

As a classroom activity and with some guidance from the teacher, students can themselves move the sliders to control the orientation of the base plane, the number of base(is) edges and the solids height.

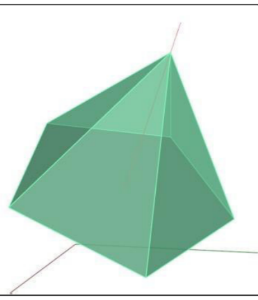
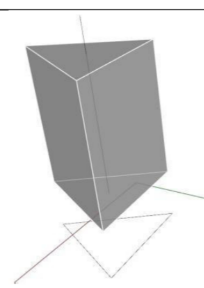
	

Fig. 7 - Pentagonal right pyramid	**Fig. 8 - Triangular right prism with its horizontal projection**

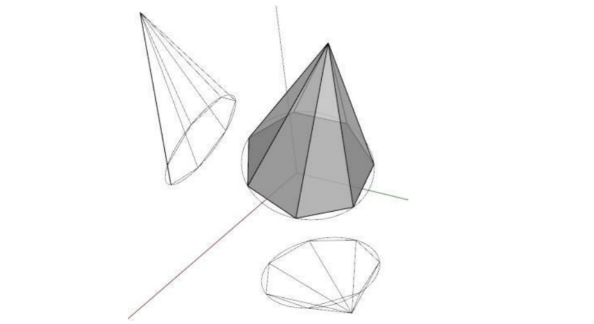
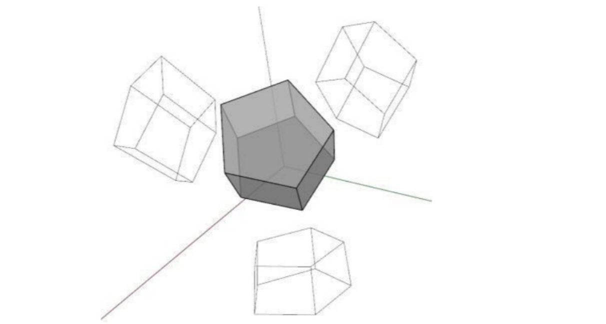
	

Fig. 9 - Heptagonal right pyramid with two orthogonal projections	**Fig. 10 - Pentagonal right pyramid with three orthogonal projections**

CONCLUSIONS

Considering the powerful digital tools nowadays available, there should be no reason for the educational environment not to explore them to create specific resources that can simplify the understanding of the concepts taught in school curricula, specifically those concerning abstract geometrical concepts which may be better understood by students through the exploration of interactive and dynamic constructions, step-by-step resolutions or virtual three-dimensional models.

It was our intention to justify the need to rethink some educational practices and to demonstrate that the exploration of digital resources by teachers and students is nowadays more emergent than ever.

> "To be a teacher does not mean simply to affirm that such a thing is so, or to deliver a lecture, etc. No, to be a teacher in the right sense is to be a learner. Instruction begins when you, the teacher, learn from the learner, put yourself in his place so that you may understand what he understands and the way he understands it." (Soren Kierkegaard, The Point of View for my Work as an Author, 1848)

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