

NEXUS 2018 Architecture and Mathematics

Conference Book



Dipartimento di Ingegneria dell'Energia, dei Sistemi,
del Territorio e delle Costruzioni
University of Pisa

and

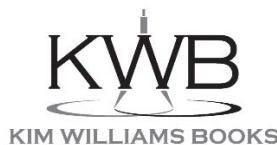
Kim Williams Books

present

Nexus 2018
Architecture and Mathematics
Conference Book

edited by

Kim Williams and Marco Giorgio Bevilacqua



Nexus 2018: Relationships Between Architecture and Mathematics
11 – 14 June 2018, Pisa, Italy

Organised by:

Dipartimento di Ingegneria dell'Energia,
dei Sistemi, del Territorio e delle
Costruzioni (DESTEC) University of Pisa



With the Patronage of:

UID – Unione Italiano Disegno



Scientific Committee

Kim Williams, Chair (Italy)
Marco Giorgio Bevilacqua, Chair (Italy)
Sylvie Duvernoy, Secretary (Italy)
Kenza Boussora (Algeria)
Ahmed Elkhateeb (Egypt)
Alessandra Capanna (Italy)
Hooman Koliji (USA)
Cornelia Leopold (Germany)
José Calvo López (Spain)
Anna Marotta (Italy)
Michael Ostwald (Australia)
Mine Özkar (Turkey)
Antonia Redondo Buitrago (Spain)
Michela Rossi (Italy)
Roberta Spallone (Italy)
Stephen R. Wassell (USA)
João Pedro Xavier (Portugal)
Maria Zack (USA)

Published by:

Kim Williams Books
<http://www.kimwilliamsbooks.com>

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, re-use of illustrations, recitation, broadcasting, reproduction on microfilms or in any other ways and storage in data banks. For any kind of use permission of the copyright owner must be obtained.

Copyright © Kim Williams Books 2018
ISBN 978-88-88479-47-7

Cover image: Nexus 2018 Logo designed by Ivan Mechkunov

Ph.D. Day Scientific Committee

Michael Ostwald – Chair (Australia)
Marco Giorgio Bevilacqua (Italy)
Sylvie Duvernoy (Italy)
Cornelia Leopold (Germany)
Anna Marotta (Italy)
José Calvo López (Spain)
Michela Rossi (Italy)
Stephen R. Wassell (USA)

Organising Committee in Pisa

Marco Giorgio Bevilacqua
Stefania Landi
Lucia Giorgetti
Alessandro Ariel Terranova

CONTENTS

Monday, 11 June 2018

Perspective, Space, Dimension, Geometry

13 CORNELIE LEOPOLD. The Development of the Geometric Concept of Relief Perspective

19 AGOSTINO DE ROSA, ALESSIO BORTOT. Hunched Curves in the Vatican: The Vestibule Arch of the Pio Clementino Museum, Between Stereotomy and Anamorphosis

25 JOÃO PAULO CABELEIRA. Deconstructing the Imaginary Space of a Quadratura

31 GIUSEPPE D'ACUNTO. Notes on Oblique Space

37 SNEZANA LAWRENCE. Lost and Found: Some Mathematical Messages from Renaissance Tuscany to a 21st-Century Teacher

43 ALESSANDRA CAPANNA. The Four-Dimensional House Theorem

49 COSIMO MONTELEONE. The Mathematical Space of Daniele Barbaro

55 LAURA CARLEVARIS. N-Dimensional Perspective: The Mathematics behind the Interpretation of Ancient Perspective

61 STEFANO CHIARENZA. Peter Nicholson and the First Interpretation of Greek Architectural Mouldings as Sections of a Cone

67 RIZAL MUSLIMIN. A Grammatical Investigation of Utzon's Spherical Schema Evolution

73 ANTONIA REDONDO BUITRAGO. On Polygons, Set Squares and Mudéjar Carpentry

79 RADOSLAV ZUK. The Visible and Invisible Geometries of Venice

Tuesday, 12 June 2018

Historical Analysis

87 FRANCESCA FATTA, DOMENICO MEDIATI. The Design of Roman Mosaics in North Africa and their Geometric References

93 ASLI AGIRBAS. Algorithmic Decomposition of Geometric Islamic Patterns: A Case Study with Star Polygon Design in the Tombstones of Ahlat

99 BERNARD PARZYSZ. What We Can Learn from Roman Geometric Mosaics about the People Who Made Them

105 JOHN KENDALL HOPKINS. A Unified Schema of the Façade of San Miniato Al Monte: The Simultaneity of Interlocking Symbolic Harmonic, Irrational and Perfect Numbers

111 SANAZ AHMADZADEH SIYAHROOD, ARGHAVAN EBRAHIMI, MOHAMMADJAVAD MAHDAVINEJAD. Application Of Cubit–Gaz and Shape Grammar in Architecture Plan Design

117 ATHANASSIOS ECONOMOU. The Six Vitruvian Principles of Architectural Design Reframed within Contemporary Computational Design Discourse

123 ROBERTA SPALLONE, MARCO VITALI. *Regola* and *Licentia* in the *Extraordinario Libro* by Sebastiano Serlio

129 ORNELLA ZERLENGA, VINCENZO CIRILLO. *Della pianta delle Scale* of Guarino Guarini

135 ANNA MAROTTA, URSULA ZICH, MARTINO PAVIGNANO. Theories and Approaches in Fortifications Design: The Figure of Gaspare Beretta

141 STEFANO BRUSAPORCI, PAMELA MAIEZZA, GIANFRANCO RUGGIERI. A Reflection on the Cistercian *Bernhardinischer Grundtypus*

147 DANIELE CALISI, MATTEO MOLINARI. Giuseppe Valadier's Urban Layout for Piazza del Popolo in Rome

153 ISABELLA FRISO, ANDREA GIORDANO. The Design Process in the Salk Institute by Louis I. Kahn

159 VINCENZO BAGNOLO, ANDREA PIRINU, MARCELLO SCHIRRU. Geometrical Design Algorithms in Nineteenth-Century Prisons: The case of the *Rotunda* in Tempio Pausania

165 JOÃO PEDRO MARQUE SÊCO DIAS CARMONA. Urban Morphology of Geometric Pattern in the Villa Imperial de Petropolis

171 ARTURO GALLOZZI, MICHELA CIGOLA. Considerations on the Representations of the Analemma in Renaissance Editions of Vitruvius's *De Architectura*

Wednesday, 13 June 2018

Contemporary Analysis, Structures, Techniques of Design, Algorithms, Rule-Based Design

179 DENISE ULIVIERI, LUCIA GIORGETTI, BENEDETTA TOGNETTI. Vittorio Giorgini Spatiology-Morphology Architect: From 'Curved Systems' to 'Conventional Systems'

185 MICHAEL J. OSTWALD, MICHAEL J. DAWES. An Isovist Analysis of Frank Lloyd Wright's Hollyhock House

191 MICHAEL C. DUDDY. Logical Accidents: The Problem of the Inside Corner

197 ORIEL E.C. PRIZEMAN, CAMILLA PEZZICA, MARIANGELA PARISI, CLARA-LARISSA LORENZ. Function Should Follow Form: Futures for the Radiant Logic of Carnegie Public Libraries

203 SHEN GUAN SHIH, YI FENG CHANG. Composite Interlocking Structures of *SL* Strands

209 ASSUNTA PELLICCIO, MARCO SACCUCCI, ERNESTO GRANDE. A Key Nexus for Vault Systems from Lecce: Stereometric Correlation Between Shape and Structure

215 VALENTINA BEATINI. Morphology of Kinetic Structures by Means of Bar and Plate 4R-Linkages

221 MARCO HEMMERLING, CARLO DE FALCO. ArchiFold: An Educational Approach for the Integration of Mathematics in the Architecture Curriculum

227 MASSIMILIANO LO TURCO, URSULA ZICH, MARCO TRISCIUOGlio, MICHELA BAROSIO, MARIA LUISA SPREAFICO, YOSEPH BAUSOLA PAGLIERO. Algorithmic Modeling and Design of the Architectural Shape: A Didactic Experience

233 FABIO BIANCONI, MARCO FILIPPUCCI, LORENZO CICULI. The Form of Music: Experiments between Cymatics and Engineering

239 MARCO CARPICECI, FABIO COLONNESE, Toward an Algorithm of Visual Design: The Mathematical Approach of Hermann Maertens' *Optische-Massstab*

245 CETTINA SANTAGATI, FEDERICO MARIO LA RUSSA, MARIATERESA GALIZIA, EUGENIO MAGNANO DI SAN LIO. Towards a Generic Parametric Algorithm for the Geometric Investigation of Baroque Oval Plans: An Application on Sicilian Cases

251 MANUEL ALEJANDRO RÓDENAS-LÓPEZ, PEDRO GARCÍA MARTÍNEZ, PEDRO MIGUEL JIMÉNEZ-VICARIO, ADOLFO PÉREZ EGEA, MARTINO PEÑA FERNÁNDEZ-SERRANO. Parametric Design Applied to Analysis and Optimization of Spatial Deployable Structures

257 MARIE-PASCALE CORCUFF. Jules Bourgoin (1838-1908): A Forerunner of Generative Shape Grammars

263 BENAY GÜRSOY, MINE ÖZKAR. *Material Shapes* and How to Compute with Them

269 MANUELA BASSETTA. Form–Formal making of Traditional Chinese Architecture

Thursday, 14 June 2018

Ph.D. Day

277 PAOLO BORIN. Geometry, Science and Meaning in the Work of Guarino Guarini

281 CRISTIAN BOSCARO. *La Manière Universelle* of Girard Desargues for the Understanding of Stereotomic Structures

285 ALEXANDRA CASTRO. The Curve in the Architecture of Herzog & de Meuron

289 RAFFAELLA DE MARCO. Shapes and Models: The Survey for the Study of Structures in Historic Buildings

293 PAOLO DI PIETRO MARTINELLI. The Control of Illusory Space: The Contribution of Jacopo Barozzi da Vignola and the Anteroom of the Council in Palazzo Farnese at Caprarola

297 MARYAM DOROUDIAN, MOHAMAD REZA BEMANIAN, MOHAMAD JAVAD MAHDAVINEJAD. Exploring of Topological Architecture: a Review of Topology Influence on Architecture

301 WILLEM GYTHIEL, MATTIAS SCHEVENELS, DIRK HUYLEBROUCK. Generating Geodesic Grid Structures by Equally Subdividing Spherical Arc Segments

305 DIOGO PEREIRA HENRIQUES. Envisioning Future Public Spaces: Experiments in Co-Creation and Evaluation of Urban Visions

309 STEFANIA LANDI, ORIOL DOMÍNGUEZ MARTÍNEZ. Modularity in Ancient Grain Storage Systems: Historical Overview and In-Depth Analysis of Moroccan Fortified Granaries

313 ELISABETTA POZZOBON. Religious Architectural Heritage Losing Its Functions: Strategies to Mitigate the Problem and Provide New Value through Geographical Context Analysis

317 MAYCON SEDREZ. Complex or Complications? Fractal Geometry in Architecture and Urban Design

321 JAKUB ŚWIERZAWSKI. Curvilinearity in Architecture: Historical and Contemporary Ideas and Examples

325 VERA VIANA. Architectonic Tessellations as Constructive Modules

ARCHITECTONIC TESSELLATIONS AS CONSTRUCTIVE MODULES

*Vera Viana*¹

Introduction

All the cells in a uniform solid tessellation (or honeycomb) are uniform polyhedra that fit together to “fill all space just once, so that every face of each polyhedron belongs to one other polyhedron” (Coxeter 1973: 68). All the vertices are equally surrounded and superimposable under symmetries onto any other. The enumeration of polyhedra that fill space is an open problem in mathematics with “no finite answer” (Grünbaum & Shephard 1980: 966) but, if we restrain to convex regular-faced polyhedra, we conclude that only 28 possibilities in which space can be uniformly tessellated exist (Grünbaum, 1994: 49). A didactic experiment on the subject of 3D modelling with first-year students of architecture in 2017 developed into the exploration of solid tessellations as constructive modules. This presentation intends to illustrate some of its outcomes.

The Research

Theoretical framework

In 1905, Alfredo Andreini enumerated 23 possibilities to uniformly close-pack convex regular-faced polyhedra (which, in fact, were only 22, since one was mistakenly considered as uniform (Grünbaum 1994: 49)). Authors such as Keith Critchlow (1969), Robert Williams (1972, 1979) and Peter Jon Pearce (1978) addressed this theme but failed to consider the complete set of convex uniform honeycombs. Until the end of the 20th century, “mathematical literature was abundant with incomplete lists” of the uniform partitions on three-dimensional space (Deza & Shtogrin 2000: 1). The enumeration of convex uniform honeycombs would be accomplished only by Norman Johnson in 1991 and later confirmed by Branko Grünbaum (1994). 13 of these solid tessellations are considered as analogues to Archimedean plane tessellations and have been categorised by Conway, Burgiel and Goodman-Strauss (2008: 292) as “Architectonic tessellations, ... because Architectonics is the theory of structural design and because its beginning reminds us of Archimedes”. Given that these 13 (from which we exclude the tessellation outlined by stacks of equal cubes) are not merely piles of extruded planar tessellations and remain still relatively unexplored in architectural design, they were selected as leitmotiv for our didactic experiment with undergraduate students of architecture. Examples of comparable polyhedral juxtapositions in architectural design (excluding the familiar cuboids) may be found in the works of Alexander Bell, Buckminster Fuller, Robert le Ricolais, Louis Khan, Jean-Francois Gabriel (all of whom explored

¹ Ph.D. Program in Science and Technology Didactics (Mathematical Sciences), Universidade de Trás-os-Montes e Alto Douro (UTAD). Integrated Researcher at Centre for Studies of Architecture and Urbanism, Faculdade de Arquitectura da Universidade do Porto, Portugal, veraviana@veraviana.net.

the tessellation outlined by regular tetrahedra and octahedra); Zvi Hecker's *Synagogue in the Negev Desert* (1967-1969) that combines cuboctahedra, truncated octahedra and truncated tetrahedra; Giancarlo Mazzanti's *Forest of Hope* (2011) in which irregular truncated octahedra combine to define a structure resembling a canopy of trees; and, among others, Peter Jon Pearce's *Curved Space Structures* (1970s) that combine convex polyhedral forms in modular structures to serve as labyrinths in children's playgrounds. Interesting examples can also be found in the concept of topological interlocking developed by the Institute of Material Science, Technical University Clausthal, in which polyhedra and osteomorphic blocks interlock to outline mutually constrained planar configurations (Tessmann 2012: 1).

A teaching experiment with space-filling systems

The subject of polyhedra is one of the most interesting through which students might be introduced to 3D modelling software, given not only its importance for the development of students' spatial literacy, but, also, their tangibility as geometrical objects connected to several branches of knowledge. In this regard, Baracs (1998: 120) denotes that "if you can create an imagery which is movable, transformable, which you can manipulate, that is the best start for imagination and creation." Although this reference is addressed to physical models, we believe the same applies to their virtual counterpart, especially if students are given the opportunity to model polyhedra themselves from the ground-up, with the additional possibility of combining them in architectural projects as modular habitable spaces or structural spaceframes.

The interesting potential of uniform honeycombs as spatial structures, repetitive construction modules or stereotomic blocks and its absence from the *curricula* of the first year at the Faculdade de Arquitectura da Universidade do Porto led us to address this topic in 3 lectures, proposing students to model and manipulate virtual and physical models of uniform convex polyhedra. Subsequently, students were introduced to the concept of Voronoi cells and primary paralelohedra, some of which were modelled and combined in spatial arrays to outline convex honeycombs, so their potential as isogonal structural frameworks could be recognized.

The following task combined different sets of convex uniform polyhedra and the possibility of juxtaposing them face-to-face, to which the students' spatial skills and mental rotation abilities were particularly important. Subsequently, students became acquainted with the 28 convex uniform tessellations and conceived, in collaborative work, spatial modular structures for a specific location in *campus*. The idea was to creatively explore an architectonic tessellation and conceive a constructible modular composition, taking advantage, as Burry & Burry propose, of "the expressive potential of imperfection and of breaking rules often exploited in built work that draws on mathematical ideas of composition" (2012: 80). Figs. 1 and 2 depict examples of models from different groups of students.

This experiment stresses out the significance of polyhedral geometry in Architecture, Arts, Design and Engineering *curricula* and, in particular, the subject of space-filling polyhedra and spatial tessellations, and its contribution for the development of students' geometrical knowledge and spatial thinking.

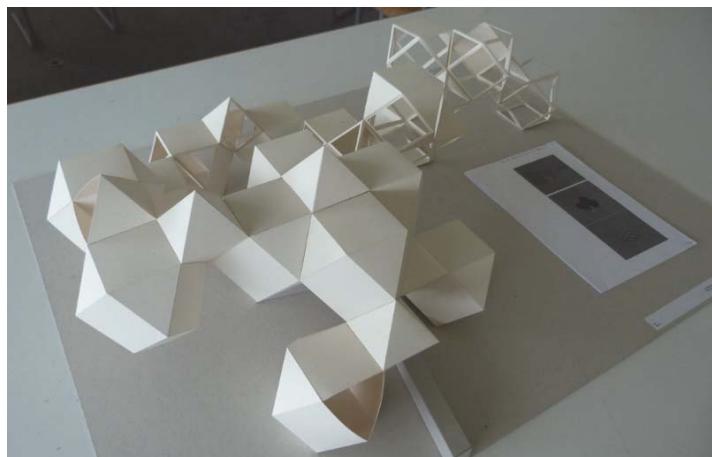


Fig. 1.

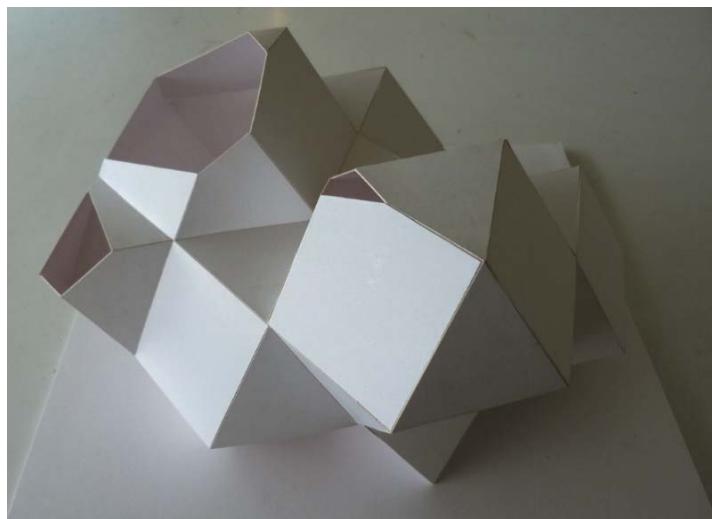


Fig. 2.

In this regard, the importance of a good 3D modelling software as pedagogical tool is not to be overlooked, without whom the possibility of operating “directly” in space to model polyhedra and combine them as modules would be less feasible or even impracticable. Further developments on this teaching experiment will propose students to conceive structural frameworks through algorithmic modelling, 3D printing and prototyping.

Conclusion

Our primary purpose with this investigation was not to introduce Students into polyhedra with the purpose of learning the 3D modelling software, but quite the contrary, to use the software as a strategy for students to learn polyhedral geometry. The procedures through which students solved the problems proposed allowed them to articulate their mathematical skills and geometrical knowledge, but further analysis

of the work developed so far will allow us to understand if the spatial literacy of students advanced any further from the starting point.

Acknowledgments

The author would like to express her gratitude to Professor João Pedro Xavier for the amazing opportunity of working this subject with his first-year students; to Professors Alexandra Castro and José Pedro Sousa who did an outstanding job of guiding students and worked with them in the accomplishment of their works, and to the students themselves, who worked in this commitment very actively and with great interest.

Assignment co-financed by the European Regional Development Fund (ERDF) through the COMPETE 2020—Operational Programme Competitiveness and Internationalization (POCI) and national funds by the FCT under the POCI-01-0145-FEDER-007744 project.

References

Andreini, A. 1905. *Sulle reti di poliedri regolari e semiregolari e sulle corrispondenti reti correlative*, Mem. Società Italiana della Scienze, Ser.3, 14 75–129. Retrieved from <http://media.accademiaxl.it/memorie/S3-VXIV-1907/Andreini75-129.pdf> in October 2017.

Baracs, J. 1988. Spatial Perception and Creativity. In: *Shaping Space: A Polyhedral Approach*, M. Senechal & G. M. Fleck, eds., pp. 118-132. Boston: Birkhäuser.

Burry, J. & M. Burry. 2012. *The New Mathematics of Architecture*. London: Thames & Hudson.

Conway, J., H. Burgiel, and C. Goodman-Strauss. 2008. Architectonic and Catoptric Tesselations. In: *The Symmetries of Things*, 292–298. London: CRC Press.

Coxeter, H. 1973. *Regular Polytopes*. New York: Dover Publications.

Critchlow, K. 1969. *Order in Space: A Design Source Book*. London: Thames and Hudson.

Deza, M. & M. Shtogrin. 2000. Uniform Partitions of 3-Space, their Relatives and Embedding. *European Journal of Combinatorics* 21(6); 807-814. DOI 10.1006/eujc.1999.0385

Grünbaum, B. 1994. Uniform tilings of 3-space. *Geombinatorics* 4: 49-56.

Grünbaum, B. & G. Shephard. 1980. Tilings with congruent tiles. *Bulletin of the American Mathematical Society* 3(3): 951-974. DOI 10.1090/s0273-0979-1980-14827-2

Pearce, P. 1978. *Structure in Nature as a Strategy for Design*. Cambridge: MIT Press.

Pearce, P. and A. Pearce. 2015. *Curved Space Diamond Structure: Origins: Geometry and Natural Structure*. CreateSpace Independent Publishing Platform.

Williams, R. 1979. *The Geometrical Foundation of Natural Structure: A Source Book of Design*. New York: Dover Publications.

Tessmann, O. 2012. Topological Interlocking Assemblies. *Physical Digitality. Proceedings of the 30th International Conference on Education and research in Computer Aided Architectural Design in Europe*, H. Achten et al., eds (eCAADe 30, vol. 2, 201-209. Brussels: Education in Computer Aided Architectural Design in Europe.